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Book review

Lagrangian Fluid Dynamics, Andrew Bennett. Cambridge University Press, Cambridge, UK (2006). 286 pp., £ 55

Two alternative approaches to describing moving fluid were developed by Euler in the mid-eighteenth century. They are traditionally referred to as Eulerian and Lagrangian description of fluid motion. The former represents the motion of fluid continuum via a velocity field while in the latter the motion of individually labelled fluid particles is observed. Leading to much simpler equations, the Eulerian description was generally regarded as the favourite, and the absolute majority of results of modern fluid mechanics are obtained by using the Eulerian approach. In many textbooks on general fluid mechanics the Lagrangian approach, if mentioned at all, is introduced as a curious example to be neglected in the rest of the text. One could hardly find reviews or monographs summarizing developments in Lagrangian fluid mechanics. Rare exceptions, for example the thematic issue of *Philosophical Transactions* on Lagrangian picture of fluid motion (*Phil. Trans. R. Soc. London A* (1990) 333, pp. 261–400), show that the Lagrangian approach was, however, not completely forgotten, but found it's specific areas of application, such as chaotic advection, turbulent dispersion or flows with interfaces, where the Lagrangian particle-following approach is the natural way to deal with complicated flow topology or to describe evolution of a material interface.

A new wave of interest in the Lagrangian description started to grow since the middle of the last century in relation to an important practical problem of interpretation of Lagrangian oceanographic and atmospheric data from freely drifting probes. Extensive development of this subject provides an example of what seems to be the most successful practical application of the Lagrangian description. And it is not surprising that the author of the first monograph specifically dedicated to Lagrangian fluid mechanics is the recognized scientist working in this area. Andrew Bennett is Professor of Physical Oceanology at the College of Oceanic and Atmospheric Sciences, Oregon State University. His main research interests are in oceanic data assimilation, inverse ocean modelling and turbulent diffusion. Professor Bennett has a long list of publications in these areas in such journals as *Journal of Geophysical Research*, *Journal of Marine Research*, *Journal of Computational Physics*, *Physics of Fluids*, etc. Two of his monographs on inverse modelling of ocean and atmosphere had been published by Cambridge University Press. In many of these works the Lagrangian approach plays an important role, and in his new monograph Professor Bennett summarizes this experience, giving an extensive account of the Lagrangian description and its applications.

The text of the book is organized in four parts: (I) The Lagrangian Formulation; (II) Lagrangian Flows; (III) Diffusion; (IV) Lagrangian Data. The first half of the book (Parts I and II) creates the core of the text giving the detailed development of the Lagrangian formulation of fluid dynamics and examples of Lagrangian solutions. This will undoubtedly be of interest to many readers of *Journal of Fluids and Structures* and indeed to any investigator of fluid dynamics who considers having the Lagrangian method among his research tools. The content of these parts is as one would expect to find in a good textbook on general fluid mechanics. Part I consists of five chapters and opens with a chapter on Lagrangian kinematics, where the fundamental concept of the particles' identity via Lagrangian labels is introduced. The specific Lagrangian notations are used throughout the book with the labelling time (release time) as an additional variable. These notations prove to be useful, helping a reader with visualizing Lagrangian ideas and distinguishing between Eulerian and Lagrangian values. Among important points covered in Chapter 1 is the labelling theorem expressing conservation of particle identity, the generalized Stokes drift, properties of Lagrangian steady flows and local Lagrangian kinematics. Chapter 2 is dedicated to Lagrangian statistics, and Lagrangian dynamics is discussed in Chapter 3, which can be considered the central point of the book. Some of the concepts discussed in Chapter 3 are Lagrangian counterparts of the familiar Eulerian principles, such as the conservation of mass, momentum and energy and Bernoulli's and Kelvin's theorems. Others, such as Cauchy–Weber integrals, Cauchy invariant, Clebsch potentials and relabelling symmetry are specifically Lagrangian. Interesting questions of applying variational principles and methods of differential geometry to Lagrangian dynamics and of the matrix form of Lagrangian equations are also addressed. Proper attention is paid to an important question of boundary conditions on rigid and co-moving boundaries. The consideration of Chapter 3 is restricted to ideal fluids, and the question of real fluids is addressed in

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Chapter 5 after a short discussion of coordinates in Chapter 4, where the invariance of Lagrangian equations to change of independent variables and the form of equations in spherical and rotating coordinates are considered.

Part II (Chapters 6–9) presents the review of analytical Lagrangian solutions. It includes both examples of Lagrangian forms of well-known Eulerian solutions such as inviscid incompressible flow around a cylinder or the boundary layer on a flat plate and purely Lagrangian solutions like Gerstner waves or Ptolemaic vortices. Some attention is also paid to Lagrangian one-dimensional gas dynamics, sound waves and hydrodynamic stability. The last chapter of Part II addresses an important question of general solvability of the Lagrangian equations of fluid dynamics. The discussion is concentrated on inviscid incompressible dynamics. Three alternative forms of equations are considered, and only one of them is found to be well posed. After this, some notes are made about compressible and viscous fluids.

The second half of the book (Parts III and IV) gives the detailed account of two particular applications of the Lagrangian approach: turbulent diffusion and Lagrangian data analysis. The scientific background of the author had obviously a determining effect on the choice of the subjects of these chapters. Although the chosen subjects provide examples of successful application of the Lagrangian description, they will not be of such general interest as are the first two chapters. Indeed, reserving more than half of a book dedicated to general Lagrangian fluid mechanics for only two specific applications seems to not be quite justified. This is either too much or too little. I believe that readers of the *Journal of Fluids and Structures* would be more interested to see the discussion of Lagrangian approaches in fluid–structure interaction, or review of numerical methods based on the Lagrangian approach. However, some sections of these two parts have value of their own, as the discussion of Lagrangian shallow-shallow theory in Chapter 16.

Finally it should be noted that the book is not easy reading. It requires from a reader a good understanding of classical fluid mechanics, strong background in advanced calculus and analysis, familiarity with tensors and most of all an ability to ''think Lagrangially''. Altogether, the first two parts of the book present a good overview of general fluid dynamics from the Lagrangian point of view. Still, they cannot be treated as a comprehensive textbook—one could not expect this from a hundred-page text. Many important points are touched very briefly. The author gives details and derivations only for the most fundamental facts. Most of the others are presented in the form of exercises with short hints and references, leaving the detailed consideration to readers. It is therefore impossible to use the book as the only source to achieve acquaintance with Lagrangian fluid mechanics. To gain sound understanding it will be necessary to work through original sources. However, albeit briefly, most of the essential points are covered in the book, and being well provided with references it can be used as a trusty guide through the subject.

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